Abstract—Lasers can operate in two regimes: continuous-wave mode or pulsed mode. In the simplest case, the former mode corresponds to monochromatic beams with Gaussian distribution of amplitudes (beam limited in space); whereas the latter mode corresponds to polychromatic beams with Gaussian distribution of frequencies (pulse limited in time). When the pulsed beams are reflected and refracted in different types of interfaces, they undergo peculiar distortions that bear some parallelism with those found for beams limited in space. These effects, as shown in a previous work, correspond to time delay (first order) and change of pulse width (second order). The distortions are clearly limited by the principle of causality and their interpretation, while not straightforward, emerges clearly when the associated fields are expressed in magnitude and phase. Since the analytical expressions are not simple even for the case where the pulse is transmitted through a single layer of linear, homogeneous, isotropic and transparent material, it makes it difficult to solve the inverse problem. In this work, we present an alternative analytical development that makes it possible to explicitly determine these distortion effects when a pulse impinges normally on a transparent isotropic layer immersed in a medium of analogous characteristics.

Keywords: Gaussian pulses; Geometric Optics; Phase shift

I. INTRODUCTION

When a 2D finite-width monochromatic beam undergoes reflection and refraction at a planar interface between two isotropic dielectric media, it is deformed. This deformation is due to the fact that a finite beam of light is composed of a superposition of a large number of plane waves that propagate at slightly different angles of incidence with respect to the central plane wave.

Under the paraxial condition, this leads to a small correction in the Fresnel equations, producing small effects that modify the beam. The first and second order effects are lateral displacement or Goos-Hänchen effect and angular shift (first order effects) and change of focal point and change of width (second order effects). These have been studied through different approaches as energy considerations [1] [2], superposition of two or more waves with different directions of propagation [3] [4] [5] [6] [7] or moment theory [8] [9]. Each of these methods have contributed to the knowledge and determination of the different non-geometrical effects.

Nevertheless, Gaussian or quasi-Gaussian beams have been the most studied space-limited beams because they are the lowest solution of the paraxial wave equation. In order to obtain analytical expressions of these effects, usually the Tamir’s generalized method (TGM) is used. This method, an extension of the Tamir’s method [5], refers to the classic
model of multiple reflections on the interfaces of an isotropic dielectric layer. The amplitudes of the successive terms of the expansion rapidly decrease, i.e., it is a rapid convergence method.

Chromatic dispersion is present in practically all optical materials and is manifested as a dependence of the phase velocity on the frequency or wavelength of light inclusive in transparent media. As known, the group velocity is the velocity that characterizes optical pulses either in dispersive or non-dispersive media as well. An optical pulse propagates at the group velocity.

In the propagation, reflection and transmission of the real Gaussian pulses there can be two types of deformations: temporal and spatial. The latter corresponds to the distortion that occurs with Gaussian beams (i.e. just spatially limited) [10].

The propagation and transformation of laser pulses are fundamental problems in the fields of laser technique, optical communication, optical information processing, etc. Many works have focused on propagation through dispersive and anisotropic media [11] [12] [13]. However, there are no studies about the distortion that occurs when a pulse (only limited in time) is reflected or transmitted through non-dispersive dielectric interfaces. In this work we study the distortions that occur on a Gaussian pulse just limited in time when it is reflected or transmitted on a single layer of non-dispersive material using an analytical development alternative to TGM. The alternative method applied to these type of pulses is a generalization of the so call moment theory of light beam propagation [14] [15] [16]. Finally the results obtained with both methods are compared.

II. INCIDENT ELECTRIC FIELD OF A GAUSSIAN PULSE

Consider a Gaussian pulse, centered on a mean frequency \( \omega_0 \) and spectral width \( \sigma \), which propagates in vacuum that impinges normally on a parallel plate of refractive index \( n \) and thickness \( d \) as shown in the figure 1. The electric fields corresponding to the incident pulse \( \vec{E}_I \) are given by

\[
\vec{E}_I(x,t) = \frac{\vec{E}_0}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} e^{-\frac{(\omega - \omega_0)^2}{2\sigma^2}} e^{i\omega(x/c-t)} d\omega (1)
\]

The integral of Eq. (1) is solved by taking \( \Omega = (\omega - \omega_0)/\sigma \) and using the series expansion of \( e^{iz} = \sum_{j=0}^{\infty} (-i)^j z^j/j! \), replacing in Eq. (1) we obtain:

\[
\vec{E}_I(x,t) = \vec{E}_0 e^{i\omega_0(x/c-t)} \sum_{k=0}^{\infty} \frac{(-i)^j}{j!} \sigma^j(t - x/c)^j Q_j \quad (2)
\]

with

\[
Q_j = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{\sigma^2}{2} \Omega^2} d\Omega
\]

The term \( Q_j \) of the Eq. (2) is the central moment of the standard normal distribution \( \bar{E}(\Omega^n) \). By the symmetry of the normal function, the terms of odd \( j \) are canceled and the moments of \( j \) pairs are calculated following the equation:

\[
\bar{E}(\Omega^{2k}) = (2k)! / 2^k k!.
\]

Replacing in Eq. (2), we obtain the electric field for the incident Gaussian pulse:

\[
\vec{E}_I(x,t) = \vec{E}_0 e^{i\omega_0(x/c-t)} e^{-\frac{\sigma^2}{2}(x/c-t)^2} \quad (3)
\]

III. REFLECTED AND TRANSMITTED ELECTRIC FIELD OF A GAUSSIAN PULSE

Assuming that the incident pulse impinges the surface normally, the electric fields corresponding to the reflected \( \vec{E}_R \) and transmitted \( \vec{E}_T \) pulses are given by

\[
\vec{E}_R(x,t) = \frac{\vec{E}_0}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} R(\omega) e^{-\frac{\sigma^2}{2} \omega^2} e^{-i\omega(x/c+t)} d\omega \quad (4)
\]

\[
\vec{E}_T(x,t) = \frac{\vec{E}_0}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\inf} T(\omega) e^{-\frac{\sigma^2}{2} \omega^2} e^{i\omega(x/c-t)} d\omega \quad (5)
\]

where \( R(\omega) \) and \( T(\omega) \) are the reflection and transmission coefficients. An analytical expression of the fields can be obtained by approximating these coefficients by a polynomial series around the average frequency \( \omega_0 \):

\[
S(\omega) = S(\omega_0) + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{\partial^k S}{\partial \omega^k} \bigg|_{\omega_0} \Delta \omega^k \quad (6)
\]

where \( S \) is \( R \) or \( T \) and \( \Delta \omega = \omega - \omega_0 \). Replacing Eq. (6) in Eq. (4) and using the same calculation procedure as for the incident field, we obtain the reflected electric field:

\[
\vec{E}_R(x,t) = \vec{E}_0 e^{-i\omega_0(x/c+t)} \left( R(\omega_0) e^{-\frac{\sigma^2}{2}(x/c+t)^2} + A_2 \right) \quad (7)
\]

with

\[
A_2 = \sum_{k=1}^{\infty} \frac{i^k}{k!} \frac{\partial^k R}{\partial \omega^k} \bigg|_{\omega_0} \sum_{j=0}^{\infty} \frac{(-i)^j (x/c + t)^j}{j!} \sigma^{k+j+1} \bar{E}(\Omega^{j+k})
\]

using the expression of the moments we can obtain a simpler expression for \( A_2 \):

\[
A_2 = \sum_{k=1}^{\infty} \frac{i^k}{k!} \frac{\partial^k R}{\partial \omega^k} \bigg|_{\omega_0} \frac{\partial^k}{\partial \omega^k} e^{-\frac{\sigma^2}{2}(x/c+t)^2}
\]
In the same way, replacing Eq. (6) in Eq. (5) and using the same procedure the transmitted field is written as

\[ \vec{E}_3 (x, t) = \vec{E}_0 e^{i \omega_0 ((x-d)/c-t)} \left( T(\omega_0) e^{-\sigma^2 (x-d)/c-t) + A_3} \right. \]

with

\[ A_3 = \sum_{k=1}^{\infty} \frac{i^k}{k!} \left. \frac{\partial^k R}{\partial \omega^k} \right|_{\omega_0} \frac{\partial^k R}{\partial \omega^k} e^{-\sigma^2((x-d)/c-t)^2} \]

IV. COMPARISON WITH THE RESULTS OBTAINED BY MEANS OF TAMIR’S GENERALIZED METHOD

In the Tamir’s generalized Method (TGM) the logarithm of the reflection and transmission coefficients can be replaced by their second order approximations around the mean frequency [17]

\[ \ln S(\omega) = \ln S(\omega_0) + \frac{\partial \ln S}{\partial \omega} \Delta \omega + \frac{1}{2} \frac{\partial^2 \ln S}{\partial \omega^2} \Delta \omega^2 \]

Introducing Eq. (9) in Eq. (4) and integrating we obtain

\[ E_3^{TGM}(x, t) = \vec{E}_0 \frac{R(\omega)}{\sigma} e^{-i \omega_0 (x/c+t)} e^{-\frac{\sigma^2}{2} (-(x/c+t)+\tau_3)^2} \]

where we define

\[ \tau_3 = -i \frac{\partial \ln R}{\partial \omega} \bigg|_{\omega_0} \quad \text{and} \quad \sigma_3^2 = \frac{\sigma^2}{1 - \sigma^2 \frac{\partial^2 \ln R}{\partial \omega^2}} \bigg|_{\omega_0} \]

Eq. (10) show that the pulse suffers a first order effect, corresponding to a complex time delay \( \tau_2 \) and a second order effect, corresponding to a complex change of half-width \( \sigma_2 \).

To compare both methods we truncate the series from Eq. (7) to second order (\( k = 2 \))

\[ E_2 (x, t) = \vec{E}_0 e^{-i \omega_0 (x/c+t)} e^{-\frac{\sigma^2}{2} (x/c+t)^2} P_2(u_2) \]

with \( u_2 = \sigma (x/c+t) \) and

\[ P_2(u) = R(\omega_0) - i \sigma \left. \frac{\partial R}{\partial \omega} \right|_{\omega_0} u - \frac{1}{2} \sigma^2 \left. \frac{\partial^2 R}{\partial \omega^2} \right|_{\omega_0} (u^2 - 1) \]

When comparing expressions Eq. (7) and Eq. (10) it is not possible to determine if both expressions are similar.

Figure 2 shows incident pulse and reflected pulse calculated using equations (7) and (10) for \( \omega_0 = 1.77 \times 10^{13} \) s\(^{-1} \); \( \sigma = \omega_0/500 \) assuming a plate of thickness \( d = 22 \mu m \); and refractive index \( n = 1.33 \); calculated for \( x = 0 \) as a function of the dimensionless time \( \omega_0 t \). The curves calculated using both approximations show a similar shift, for the case of the TGM approximation the shift of \( \omega_0 \tau \approx 91 \), while in the presented development it is \( \omega_0 \tau \approx 98 \). However, the half-width of the half-height pulse (\( \Delta \)) in both cases coincides with \( \omega_0 \Delta = 607 \) being 1.03 times the width of the initial pulse.

The electric field of the transmitted pulse calculated using the TGM approximation is calculated in a similar way by replacing equation Eq. (9) in Eq. (5)

\[ E_3^{TGM} (x, t) = \vec{E}_0 \frac{S(\omega)}{\sigma} e^{-i \omega_0 ((x-d)/c-t)} e^{-B(t)} \]

with

\[ B(t) = \frac{\sigma_3^2}{2} (-(x-d)/c+t + \tau_3)^2 \]

where we define

\[ \tau_3 = -i \frac{\partial \ln T}{\partial \omega} \bigg|_{\omega_0} \quad \text{and} \quad \sigma_3^2 = \frac{\sigma^2}{1 - \sigma^2 \frac{\partial^2 \ln T}{\partial \omega^2}} \bigg|_{\omega_0} \]

In order to compare both methods we developed the Eq. (8) to second order

\[ E_3 (x, t) = \vec{E}_0 e^{-i \omega_0 ((x-d)/c-t)} e^{-\frac{\sigma^2}{2} (x-d)/c-t)^2} P_3(u_3) \]

\[ P_3(u) = T(\omega_0) - i \sigma \left. \frac{\partial T}{\partial \omega} \right|_{\omega_0} u - \frac{1}{2} \sigma^2 \left. \frac{\partial^2 T}{\partial \omega^2} \right|_{\omega_0} (u^2 - 1) \]

In Fig. 3 shows the incident pulse and the transmitted pulse calculated using equations (15) and (13) calculated for \( x = d \). In this case the curves calculated using both approximations show a similar shift \( \omega_0 \tau \approx 93 \), however the pulse half-width in both cases coincides \( \omega_0 \Delta = 588 \) being the same as that of the initial pulse.

V. CONCLUSION

We show that Gaussian pulses, when reflected or transmitted on a single layer, exhibit peculiar distortions. We determine expressions for the first and second order effects: time delay and pulse width. The method used (method of moments) is an alternative to the generalized Tamir’s method and extended to pulses limited in time. Although the distortions are very low at optical frequencies and begin to be noticeable at frequencies of microwaves or less we can see that the results obtained do not differ numerically from each other. Nevertheless, from the method of moments it is not simple to extract explicit expressions for such
deformations; but instead the TGM provides explicit expressions, simple to calculate. In all cases the major analytical difficulty is to calculate explicit expressions of the real and imaginary parts of the fields (or of their module and phases) when dealing with complex interfaces (several interfaces).

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REFERENCES


